What is the room for guessing in metacognition? Findings in mathematics problem solving based on gender differences

Introduction. Understanding the interaction of metacognitive strategies with guessing in problem-solving is a focus point in educational psychology, especially in mathematics education, especially with gender differences. These provide different nuances in understanding students’ cognitive dynamics in guessing and metacognitive strategies. These interactions must be explained in depth to understand the unique phenomenon of second-guessing that sometimes arises in metacognitive activities based on students’ gender. This research aims to reveal the process of metacognition and students’ guessing thinking in solving mathematical problems. This research seeks to contribute to an empirical of metacognitive processes and their practical implications for educators and curriculum designers by uncovering these activities.

Study participants and methods. The participants in this study were (30) 12th-grade high school students, consisting of (16) female students and (14) male students who had studied mathematics with statistical material. The primary method in this research is a case study with a qualitative descriptive approach. The in-depth interview technique using the MAI protocol is based on the results of mathematical problem-solving to explore information about students’ natural metacognitive processes according to what they think when working on problems. Semi-structured interviews used the Indonesian language to eliminate the influence of differences in regional language proficiency levels as much as possible.

Results. The results of interviews on mathematical problem-solving tasks show a ‘guessing’ strategy in students’ metacognitive activities. However, there is a striking difference between female and male students’ use of the ‘guessing’ strategy when solving mathematical problems. Female students use the ‘guessing’ method more often than male students; even the ‘guessing’ method used by male students cannot solve the problem, thus changing the guessing strategy to estimation. This means that most of the students’ answers used the guessing method. These findings underscore the dynamic nature of the cognitive processes used by students, revealing diverse interactions between metacognitive strategies and guessing during problem-solving efforts.

Conclusion. This research shows that rather than standing in isolation, guessing has a place within metacognitive processes, with metacognitive regulation guiding and shaping the deliberate application of guessing in mathematical problem-solving contexts. This can be a factor for reconsidering the perceived dichotomy between precision-driven methodologies and intuitive guessing.

Keywords: guessing, metacognition, mathematics, problem-solving, gender, case study

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INTRODUCTION

Essential skills in the 21st century are still relevant to the four pillars of education according to UNESCO which include learning to know, learning to do, learning to be and learning to live together [1]. In mathematical problem-solving, the complex interplay between cognitive processes and strategic approaches has long been an object of research [2]. Among these strategies, the role of metacognitive knowledge and regulation of one's metacognition has emerged as a focal point in understanding how one deals with challenging mathematical problems [3]. However, an exciting and often debated question remains within this landscape: Is there room for guessing in solving math problems? This question has been answered by several previous studies, such as research by Habibah et al. [4] which said guessing is used by students when solving geometry problems. Mathematical problem-solving certainly has a complex solution, cannot be solved quickly, and requires a deep cognition process; several studies have explained that problem-solving will require metacognition strategies. Metacognitive strategies become an interesting discussion when guessing is involved. Is guessing still relevant to use when students' metacognition is running? We now focus on the question: What is the room for guessing in metacognition?

Mathematics, often considered the domain of precision and accuracy, juxtaposes the notion of guessing into a term usually associated with uncertainty and lack of precision. However, the concept of guessing in mathematics is not only associated with uncertainty, as guessing games with lies are equivalent to an essential mathematical object called error-correcting codes [5]. The role of metacognitive activities, particularly on "guessing" strategies, in mathematical problem-solving among students is a critical area of study. Metacognitive activities involve awareness and regulation of one's thought processes, which play an essential role in mathematical problem-solving. In the early stages of mathematical problem-solving, metacognitive knowledge, and planning play an essential role in preventing students from taking a trial-and-error approach and allowing students to use prior knowledge strategically by determining what information is given and what is asked [6]. Moreover, amidst the rigor of mathematical procedures, there are instances where intuitive or speculative leaps play a role in problem-solving. These instances, colloquially referred to as ‘guessing,’ prompted an investigation into the extent and importance of such approaches in metacognitive activities among students.

Based on relevant references, there is indeed an alignment between guessing strategies and metacognition. Metacognition, which involves awareness and understanding of one's thought processes, is closely related to guessing strategies in various learning contexts [7; 8]. The use of guessing as a strategy is often associated with metacognitive control and awareness, as learners use this strategy to compensate for information that is believed to be poorly remembered [9]. The importance of metacognitive strategies in problem-solving has been well documented, emphasizing the importance of self-regulation, monitoring, and reflection during the problem-solving process. However, explicit exploration of the manifestations of guessing, its implications and potential alignment with metacognitive activities still needs to be explored in the context of mathematical problem-solving strategies.

This qualitative research attempts to bridge this gap by exploring students' metacognitive activities and the potential emergence of guessing activities during mathematical problem-solving. Through in-depth qualitative analysis, this research aims to reveal the process of
metacognition and students' guessing thinking in solving mathematical problems. This research seeks to contribute to an empirical, theoretical understanding of metacognitive processes and their practical implications for educators and curriculum designers by uncovering these activities. The insights gained from this research may offer a different perspective on mathematics teaching and learning, potentially redefining conventional boundaries between rigor-driven methodologies and intuitive leaps in problem-solving approaches. In essence, this research seeks to navigate the complex terrain where cognitive strategies meet intuitive guessing, offering a deeper understanding of students' metacognitive activities and the potential role of guessing in mathematical problem-solving.

**THEORETICAL ISSUES**

The domain of metacognition in mathematical problem-solving has attracted significant attention in educational psychology, which emphasizes the importance of self-regulation, monitoring, and reflection in the cognitive processes involved in overcoming mathematical challenges. Metacognition, as described by Flavell [10], includes two fundamental components: metacognitive knowledge and metacognitive regulation. Metacognitive knowledge refers to an individual's understanding of cognitive processes, strategies, and task requirements. In mathematical problem-solving, this knowledge manifests in students' awareness of problem-solving strategies, such as heuristic approaches, algorithms, and problem-solving schemes. Metacognitive regulation, on the other hand, involves strategically applying and adapting these cognitive processes through planning, monitoring, and evaluation during problem-solving tasks. Schoenfeld [11] seminal work introduced the concept of metacognitive monitoring, which emphasizes the importance of students' ability to assess the effectiveness of their problem-solving strategies. These studies underscored the positive correlation between students' metacognitive skills, such as self-regulation and reflection, and their proficiency in solving mathematical problems.

Recent advances in this area have shed light on the dynamic nature of metacognition in mathematical problem-solving. The merging of technology-enhanced learning environments has paved the way for exploring metacognitive processes in digital contexts. The study by Lobczowski et al. [12] has investigated how digital platforms and adaptive learning systems can facilitate metacognitive regulation and self-assessment among students engaged in mathematical problem-solving. Research has also shown that students who use their metacognitive skills and creative thinking, such as reformulating problems, analyzing given information, and using alternative strategies, exhibit successful problem-solving abilities [13; 14]. Therefore, students need metacognition to improve their problem-solving ability when solving math problems.

Metacognitive ability is essential for students in solving mathematical problems, as it helps organize cognitive operations, manage the problem-solving process, and regulate performance [15; 16]. Guessing, often considered a spontaneous or intuitive leap in problem-solving, is a cognitive strategy with many aspects that differ from traditional systematic approaches. Although traditionally considered harmful due to its association with uncertainty, guessing takes many forms, including educated guesses, intuitive hunches, and trial-and-error approaches [17]. In mathematical problem solving, guessing emerges as a strategic yet unconventional approach that differs from algorithmic or systematic methods. The effectiveness and usefulness of guessing in problem-solving have been the
subject of extensive research. However, the efficacy of guessing in mathematical problem-solving is still debated. Gigerenzer and Gaissmaier [18] emphasized the adaptive nature of simple heuristics, suggesting that in specific contexts, guessing or heuristic approaches can outperform complex algorithms, especially when faced with uncertainty or time constraints. Despite their potential benefits, guesswork in mathematical problem-solving presents challenges, especially in educational settings. The emphasis on precision, accuracy, and step-by-step methodology in mathematics education often marginalizes intuitive or speculative approaches. Educational paradigms prioritize algorithmic mastery over intuitive leaps, potentially inhibiting students' exploration of alternative problem-solving strategies.

The intersection of metacognition and guessing in the context of problem-solving is a dynamic and exciting area of inquiry. While metacognition includes conscious awareness and regulation of cognitive processes, guessing often involves intuitive or speculative problem-solving leaps. Despite their apparent differences, these cognitive strategies share a common link in problem-solving. Metacognitive sensitivity refers to one's ability to distinguish between different performance levels, such as correct or incorrect trials [19]. Efklides' [20] research highlights the adaptive nature of metacognitive strategies, emphasizing their role in guiding and optimizing cognitive processes during problem-solving tasks. Metacognitive regulation, including planning, monitoring, and evaluation, assists individuals in selecting and adapting problem-solving strategies, which include intuitive or heuristic approaches characteristic of guessing.

Gender differences in mathematics performance have been a subject of extensive research, with scholars aiming to understand the various factors that may contribute to observed variations in problem-solving abilities between males and females. The impact of gender stereotypes on mathematics performance has been extensively examined. Hyde and Linn [21] suggested that societal stereotypes about gender roles in mathematics can affect individuals' self-perceptions and confidence in problem-solving situations. This perspective implies that sociocultural expectations may partly shape gender differences in mathematics. No statistically significant differences were found in performance nor planning time in ToL between the age and gender groups in any of the task conditions. The findings highlight the need to analyze the interaction between cognitive and emotional processing, individual differences, and task demands [22].

Several studies have explored cognitive and metacognitive differences between males and females in mathematics. Voyer and Voyer [23] examined gender differences in cognitive abilities. They found that while males outperform females in specific spatial tasks, females exhibit advantages in verbal and memory tasks. The implications of these cognitive differences on metacognition and problem-solving strategies are crucial to understanding gender disparities. Research by Mefoh et al. [24] delved into examining metacognitive strategies employed by males and females in mathematics problem-solving. They identified variations in strategy use, with females often adopting more reflective and systematic approaches, while males tended to employ heuristic and trial-and-error methods. These findings underscore the importance of considering metacognitive diversity when exploring gender differences in problem-solving outcomes.

In an educational context, integrating metacognitive and guessing strategies provides exciting implications for pedagogy. Studies by Veenman et al. [25] and Artzt and Armour-Thomas [26] emphasize the importance of fostering metacognitive environments that encourage students to explore various problem-solving strategies, including intuitive leaps or educated guesses. These studies suggest that metacognitive awareness can facilitate the
incorporation of deliberate guessing in students' problem-solving repertoires, potentially increasing their flexibility and adaptability in addressing mathematical challenges. On the other hand, guessing is a strategy that involves making educated guesses when one is unsure of the answer. While guessing can be a helpful strategy in some cases, it can also lead to errors and hinder the development of metacognitive skills. However, measures of relative metacognitive accuracy can be confounded with task performance in tasks that allow guessing, leading to biased estimates of the relationship between metacognition and task performance [27]. Despite these advances, there still needs to be more understanding of the intricate relationship between metacognitive strategies and other cognitive approaches, such as guessing, in mathematical problem-solving. The integration of guessing – a seemingly intuitive and speculative aspect – within the framework of metacognitive activities remains an under-explored area, prompting further qualitative investigation. Based on these considerations, this study sought to explore the intersection between metacognition and guessing, aiming to uncover the intricacies and potential synergies between these cognitive strategies in mathematical problem-solving among students.

**RESEARCH METHODS**

*Research design*

The case study research design with a qualitative descriptive approach allows participants to give meaning to their experiences about their thought processes, making it suitable for research that requires in-depth explanations [28]. Due to the human element involved in qualitative research, the researcher's involvement is crucial as data is mediated through 'human instruments' (participants) rather than inventories, questionnaires, or machines [29]. As researchers, we were involved in recognizing the participants' thought processes, perceptions, experiences, and personal interests in solving statistics problems. As this study is based on participants' reflections on their metacognitive knowledge and regulation after solving statistics problems, a qualitative research design with an instinctive case study approach was chosen, as descriptions of students' metacognitive activities were explored to provide rich and exciting data on the presence of guessing strategies in students' metacognition when executing problem-solving plans.

**Table 1**

<table>
<thead>
<tr>
<th>Gender Characteristics</th>
<th>Scoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feminine</td>
<td>$M \leq SM_{\text{Mf}} \land F \geq SF_{\text{Mf}}$</td>
</tr>
<tr>
<td>Masculine</td>
<td>$M \geq SM_{\text{Mf}} \land F \leq SF_{\text{Mf}}$</td>
</tr>
<tr>
<td>Androgynous</td>
<td>$M \geq SM_{\text{Mf}} \land F \geq SF_{\text{Mf}}$</td>
</tr>
<tr>
<td>Undifferentiated</td>
<td>$M \leq SM_{\text{Mf}} \land F \leq SF_{\text{Mf}}$</td>
</tr>
</tbody>
</table>

*Participants*

A total of 30 participants were involved in this study; 16 female and 14 male participants were involved in the pilot test in the selection of subjects. Participants are high school students who have experience learning statistics material. Grouping students based on gender characteristics using the Bem Sexual Role Inventory (BSRI) questionnaire (see Table
1), then students who have been grouped by gender are tested using a math ability test to

group students based on their abilities (see Table 2). From the 30 participants tested with the

statistical problem-solving task, 2 participants with the same moderate mathematics ability

different gender characteristics (one female and one male student) were selected to be

interviewed in depth using the MAI protocol based on the statistical problem-solving task.

The language used in the interview process was Indonesian to facilitate understanding of

questions and interpretation of student responses.

<table>
<thead>
<tr>
<th>Mathematical Ability Level</th>
<th>Scoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>x≥80</td>
</tr>
<tr>
<td>Moderate</td>
<td>60≤x&lt;80</td>
</tr>
<tr>
<td>Low</td>
<td>x&lt;60</td>
</tr>
</tbody>
</table>

The problem in investigation

The statistical problem we chose to address was the mean and median from the official

Mathematics textbook for grade 12 in Indonesian secondary education, which was part of

the current curriculum at the time of this study. We design this problem in a contextual,

ill-defined, and non-routine manner by containing several possible solutions that can be

used, one of which is guessing by looking at data patterns or looking at the conditions of

the problem to focus on the goal of seeing the presence of guessing strategies in students'

metacognitive processes when solving problems because the complexity of the problem

content is also a critical factor that encourages students to guess randomly [30].

The designed problem is as follows: At the Nusantara Indah High School anniversary

celebration, a coin arrangement competition was held. The competition was attended

by 12th-grade students, including Radit, Gita, Adi, Nuning, Linda, and Sambo. The first

group was followed by male participants, namely Radit, Adi, and Sambo. Radit managed

to arrange coins as high as 70 cm, Adi as high as 60 cm, and Sambo as high as 20 cm.

The second group was followed by female participants: Gita, Nuning, and Linda. Gita

managed to arrange the coins as high as x cm, Nuning as high as 30 cm, and Linda as

high as 40 cm. If the results of the coin arrangement height of the first and second

group participants are combined into one data so that the mean and median values

are the same, then determine the value of x if Gita's coin arrangement height is greater

than Sambo's coin arrangement height.

The expected solution uses the above method and is completed up to this stage. However,

one may complete a mathematical problem-solving task by following a trial-and-error

approach or guessing consciously under the control of their metacognition or randomly,

without being aware of the interpretation of the data as required by the problem. Is there

room for students to use guesswork when using metacognitive skills? Such a question

would provide a different perspective on the original problem and highlight metacognition's

significant contribution to the formulation and testing of guesses. To that end, we will find

different perspectives of cognitive activity, whether or not it is supported by metacognitive

regulation and metacognitive knowledge.
Method of data analyzing

The metacognitive activities of female and male students were explored using in-depth interviews referring to the table of metacognitive activities that we compiled according to the needs of interviews based on mathematical problem-solving tasks. The questions asked were developed based on students' problem-solving results concerning the metacognitive activity points (see Table 3).

<table>
<thead>
<tr>
<th>Components</th>
<th>Subcomponents</th>
<th>Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metacognition</td>
<td>Declarative Knowledge</td>
<td>Knowing consciously about one's own abilities that include memory, weaknesses, and strengths.</td>
</tr>
<tr>
<td></td>
<td>Procedural Knowledge</td>
<td>Know about how to apply steps in doing something.</td>
</tr>
<tr>
<td></td>
<td>Conditional Knowledge</td>
<td>Know about why and when to use the right information and procedures in doing something.</td>
</tr>
<tr>
<td>Metacognition</td>
<td>Planning</td>
<td>Realizing understanding in developing a plan to do something.</td>
</tr>
<tr>
<td></td>
<td>Monitoring</td>
<td>Being directly aware of understanding and performance of cognition in solving problems</td>
</tr>
<tr>
<td></td>
<td>Evaluating</td>
<td>Realizing the results of his thinking process by evaluating and assessing the control process and the results of his thinking in solving the problem.</td>
</tr>
</tbody>
</table>

FINDINGS

In this section of the results, we show the data distribution of the students involved in this study to be selected for the case studies and interviews. As we explained earlier, 30 participants were involved in this study; we grouped them based on gender and math ability level. Table 4 presents the gender and math ability distribution of the grouped students. For gender characteristics, we symbolize M as Masculine, F as Feminine, A as Androgynous, and U as Undifferentiated.

Distribution of mathematical ability by gender

<table>
<thead>
<tr>
<th>Mathematical Ability</th>
<th>Sex with gender characters</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male (M, F, A, U)</td>
<td>Female (M, F, A, U)</td>
</tr>
<tr>
<td>High</td>
<td>1 0 1 0</td>
<td>0 1 1 0</td>
</tr>
<tr>
<td>Moderate</td>
<td>8 1 0 0</td>
<td>9 2 0 0</td>
</tr>
<tr>
<td>Low</td>
<td>3 0 0 0</td>
<td>2 1 0 0</td>
</tr>
</tbody>
</table>

The results obtained from the implementation of mathematics ability tests and gender questionnaires are that of the 30 class students who took the TKM test, there were four students in the high category consisting of 1 masculine male, one androgynous male, one feminine female, and one androgynous female with a math ability test acquisition score greater than 80 each. There were 20 students in the medium category consisting of eight masculine males, one feminine male, nine feminine females, and two androgynous females.
with a math ability test acquisition score between 71 and 79. Six students in the low category consist of three masculine men, two feminine women, and one androgynous woman with a math ability test score of less than 70.

So based on the results of this grouping, two subjects were taken, namely one feminine female student (BSRI score: $3,80_\text{M} < 4,98_\text{F} > 4,20_\text{N}$) with moderate to high math ability (mathematic ability score: 79) hereafter referred to as Subject-1 (FS) and one masculine male student (BSRI score: $5,10_\text{M} > 4,30_\text{F} < 4,70_\text{N}$) with moderate mathematical ability tending to be high (mathematic ability score: 77) will be called Subject-2 (MS). Selecting one feminine female and one masculine male student in the moderate math ability category was done to reduce bias because the moderate math ability group was the dominant group. The data used for further analysis are the results of FS and MS's mathematical problem-solving tasks and interviews in solving statistical problems that have been given related to statistical material.

**Female students' mathematical problem-solving results (FS)**

Female students use metacognitive knowledge strategies when carrying out the problem-solving process; this can be seen in students' statements during interviews. As in this interview excerpt, female students explain the strategy used by "looking one by one" and "guessing," in this case, meaning that students focus on facts and algorithms on data that is already known in the problem. The opportunity to guess based on that fact is utilized by students to control their cognition to work on problems; this activity is procedural knowledge. This knowledge control is a link to cognition control in the 'planning' carried out in finding the value of $x$. As in the interview excerpt below, the student guessed the value of $x$ in "a number greater than 20". However, the student reaffirms the previous statement after 'monitoring' the conditions in the question; the student says there is a "next clue," which means that the value of $x$ must be greater than 70. This activity is an activity on metacognitive regulation that is interconnected with metacognitive knowledge. Students start 'guessing' with 50, but after realizing the 'clue' from the question requiring more than 70, cross out 50 and change their 'guess' to 80 (see Figure 1).

**R:** What are the mean and median values that you got?
**FS:** I found the mean 50 and median 50, sir.
**R:** For the $x$ value, did you find it one by one, or is there another way?
**FS:** Yes, I looked for it one by one, and I guessed the numbers according to this known data ...
**R:** What number did you start with?
**FS:** Only numbers greater than 20... But we see again that the next clue must be higher than Radit, so it's impossible to enter 30 with 40 or 50 because it must be higher than 70...
**R:** Are you sure about this answer?
**FS:** Yes, sir.
If seen based on the answer in Figure 1, female students immediately use the mean formula by guessing the unknown data; after finding the mean value, then female students continue to find the median value by sorting the data from smallest to largest; there are two scribbles on the numbers 20 and 50 in the data arrangement to calculate the median, the number 20 may be an omission, but the number 50 is part of the guessed number used because the operation result $90/2 = 45$ is scribbled to $100/2 = 50$. This activity reinforces a 'guessing' activity in the metacognitive process of female students. The conclusion related to the height of Gita’s coin is only on the height of the 80 cm coin. This is not wrong, but the possibility of other coin heights higher than Sambo's coin is neglected because the cognition process of students' metacognition results is focused on qualifying data only. Female students are also confident with the answers given; they 'evaluate' the results of their thinking and feel confident with their answers.

**Male students’ mathematical problem-solving results (MS)**

Male students use metacognitive knowledge strategies when problem-solving; this can be seen in student statements during interviews. As in the following interview excerpt, male students explained that the method used initially was "guessing." in this case, male students also focus on facts and algorithms on the known data in the problem but do not find the final solution according to the problem requirements. Namely, the mean and median must be the same. The opportunity to guess based on known data facts is also used by male students to control their cognition in working on problems, but they still have to achieve the goal; this activity is procedural knowledge. This knowledge control is linked with cognitive control in the 'monitoring' carried out in finding the value of $x$ because the male student's statement, "so I use the method of estimation," indicates that the male student changed the method of working from "guessing" to "estimation" because there was cognitive control from the results of 'monitoring' the goal to be achieved in accordance with the conditions specified in the problem, namely the mean and median values must be the same. This activity is a metacognitive regulation activity.

R: What is your mean score?
MS: For the male group, the mean is 50; for the female group, it's 50, too, sir.
R: How did you get the same mean of 50?
MS: For the male group, I added up all the data and then divided by 3, resulting in a mean value of 50, sir; for the female, because only 2 data are known, so I just added up the existing data so that I got 70$\times$, because the conditions must be the same so I tried to suppose the total was 150 too, so the value of 70$\times$ was operated on 150 sir, then the result was $x = 80$, so if 150 is divided by 3 it gets 50 too sir, at first I guessed the value of $x$ the result was not found sir, so I used this method of memorization.

**Figure 1** Female students’ answers

<table>
<thead>
<tr>
<th>Answer:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean: $70 + 60 + 20 + 30 + 40 + ... /$ total data</td>
</tr>
<tr>
<td>For example, I use the number 80</td>
</tr>
<tr>
<td>So, $70 + 60 + 20 + 30 + 40 + 80 / 6 = 300/6 = 50$</td>
</tr>
<tr>
<td>Median 30, 40, 60, 70, 80 = $100/2 = 50$</td>
</tr>
<tr>
<td>So, $x = 80$ cm</td>
</tr>
</tbody>
</table>

Gita's coin is 80 centimeters tall, higher than Sambo's 20 cm
R: What about the median?

MS: I used the median of single data only, sir. I arranged the data of each group, then took the middle value; if the male group was right at 60 if the female group was right at 40, then I tried to add 60 + 40 = 100, then I divided it by 2 like the median formula sir, I got 50 too, so I concluded that the height of Gita's coin was $x = 80$.

R: Are you sure about this answer?

MS: Yes, sir, I'm sure this is correct.

When viewed based on the answer in Figure 2, male students divide the group into two parts and then use the mean formula by assuming the total data is equal to 150, then substitute the $x$ value to produce the same mean value at 50. Male students continue to look for the median value by sorting the data from smallest to largest according to the group; there is a little scribble on the number 40, which is then changed to 30 in the data order to calculate the median. The number 40, which is scored, is an omission. However, the $x$ value = 80 is immediately entered into the data order to get the middle value. Male students then combined the middle values of the two groups by using the median formula again $100/2 = 50$. This activity does not corroborate the existence of guessing activities in the metacognitive process of male students because there is no visible guessing process in the answer. However, based on the interview, students consciously thought about the previous guessing strategy but did not find a solution, so they replaced it with memorization. The conclusion regarding the height of Gita's coin only at 80 cm is the same as that of female students; male students also focus on qualifying data facts only. Male students also 'evaluated' their answers and believed they were correct.

<table>
<thead>
<tr>
<th>Male group:</th>
<th>Female group:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$70 + 60 + 20 = 150$</td>
<td>$x + 30 + 40 = 70x = 150$</td>
</tr>
<tr>
<td>$= 150/3 = 50$</td>
<td>$x = 80$</td>
</tr>
<tr>
<td></td>
<td>$= 150/3$</td>
</tr>
<tr>
<td></td>
<td>$= 50$</td>
</tr>
</tbody>
</table>

Mean = 50

So, $x$ is 80

Median: 20, 60, 70 (male group)

: 20, 30, 40, 60, 70, 80 = 100/2 = 50

(female group + male group)

**Figure 2** Male students' answers
A qualitative exploration of students' metacognitive activities and their potential intersection with guessing in mathematical problem-solving based on gender differences has provided insights and answers to support and challenge some previous research. The findings underscore the dynamic nature of the cognitive processes employed by students, revealing diverse interactions between metacognitive strategies and guessing during problem-solving attempts. The results of this study show a 'guessing' strategy in students' metacognitive activities. However, there is a notable difference in female and male students' use of the 'guessing' strategy when solving mathematical problems. Female students used the 'guess' method more than male students, and even the male 'guess' method did not solve the problem. This implies that most of the female students' answers use guesses. This result is in line with research conducted by Ossai [31] and contradicts Baldiga's research [32]. Ossai's study in 2015 showed that women 'guessed' more than men when participants were not told to keep from guessing. This study tends to reason that the results may have been influenced by the fact that the research subjects were not informed about using correction to guess. Baldiga's study in 2014 showed that women and men are less willing to guess if they are at a disadvantage. This tendency may change if students are made to be cautious in guessing when solving math problems.

The analysis also showed that the participants demonstrated students' metacognitive engagement from planning and monitoring to evaluating the problem-solving approach. This finding aligns with previous research that emphasizes the adaptive nature of metacognition in optimizing cognitive processes. Focusing on facts and algorithms on known data in the problem also triggers guessing in metacognition [33]. Students continue to experience reliance on prior knowledge in mathematical problem-solving tasks, especially statistics [32; 33].

The participants' reflections informed the role of guessing as a strategic element in the problem-solving repertoire. Contrary to the conventional notion of guessing as a random or haphazard approach, the participants described instances where the guessing was deliberate and guided by the understanding of data patterns on the problem. This challenges the dichotomy between systematic approaches and intuitive guessing, which aligns with the notion of heuristics as adaptive problem-solving tools [18]. However, students' metacognitive performance tends to decline, perhaps due to the negative bias of 'guessing' [27]. So, in the future, a learning formulation and tasks are needed to build a positive correlation of 'guessing' to students' metacognitive performance in problem-solving.

The female students used metacognitive knowledge and metacognitive regulation skills that included the 'guessing' strategy in solving the math problem, and the male students also used metacognitive knowledge and metacognitive regulation that included the 'guessing' strategy, although only to a lesser extent than the female students. Although the students' answers did not use the formulated procedural steps for this math problem, these students engaged in critical and reflective thinking, similar to when working with data patterns. When asked, these students could articulate their systematic guesses and check the results of their cognition orally or in writing.
The coexistence of metacognitive strategies and guessing challenges the traditional boundaries between systematic and intuitive problem-solving approaches. This research shows that rather than standing in isolation, guessing has a place within metacognitive processes, with metacognitive regulation guiding and shaping the deliberate application of guessing in mathematical problem-solving contexts. This can be a factor for reconsidering the perceived dichotomy between precision-driven methodologies and intuitive guessing.

**RECOMMENDATION**

The insights gained from this study have practical implications for mathematics education. Creating environments that foster metacognitive awareness and encourage the incorporation of deliberate guessing can improve students' adaptability in problem-solving. Educators may consider integrating activities that explicitly encourage students to reflect on their problem-solving approaches, recognizing the value of systematic reasoning and intuitive leaps.

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**Conflict of Interest**

No conflict of interest is declared by authors.

**Author Contributions**

All authors have sufficiently contributed to the study and agreed with the results and conclusions.

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**Information about the authors**

**Kiki Henra**  
(Indonesia, Bone)  
Doctoral Candidate in Mathematics Education,  
Study Program of Mathematics Education,  
Universitas Negeri Surabaya  
Lecturer in Mathematics Education, Study Program of Mathematics Education  
Universitas Muhammadiyah Bone  
E-mail: kiki.21015@mhs.unesa.ac.id  
ORCID ID: 0000-0001-8982-8266

**I Ketut Budayasa**  
(Indonesia, Surabaya)  
Professor, PhD in Mathematics,  
Study Program of Mathematics Education  
Universitas Negeri Surabaya  
E-mail: ketutbudayasa@unesa.ac.id  
ORCID ID: 0000-0002-5066-859X

**Ismail**  
(Indonesia, Surabaya)  
Doctor in Mathematics Education,  
Study Program of Mathematics Education  
Universitas Negeri Surabaya  
E-mail: ismail@unesa.ac.id  
ORCID ID: 0000-0003-1969-5415

**Meina Liu**  
(U.S., Rochester)  
Doctoral Candidate in Educational Leadership  
University of Rochester  
E-mail: mliu50@u.rochester.edu  
ORCID ID: 0000-0002-3534-3156